Worked Example 4 (Version 1)

Design of a tied-back retaining wall to resist earthquake loading

Worked example to accompany MBIE Guidance on the seismic design of retaining structures for residential sites in Greater Christchurch (Version 2) November 2014

Tied-back retaining walls were used originally as a substitute for braced retaining walls in deep excavations. Ground anchor tie-backs were used to replace bracing struts that caused congestion and construction difficulty within the excavation. Design procedures evolved from those developed for braced excavations and are typically based on the so-called “apparent earth pressure” diagrams of Terzaghi and Peck [1967] and Peck [1969]. These diagrams were developed empirically from measurements of loads imposed on bracing struts during deep excavations in sands in Berlin, Munich, and New York; in soft to medium insensitive glacial clays in Chicago; and in soft to medium insensitive marine clays in Oslo.

These original “apparent earth pressure diagrams” were not intended by the authors to be a realistic representation of actual earth pressures against a wall but to be “…merely an artifice for calculating values of the strut loads that will not be exceeded in any real strut in a similar open cut. In general, the bending moments in the sheeting or soldier piles, and in wales and lagging, will be substantially smaller than those calculated from the apparent earth pressure diagram suggested for determining strut loads.”[Terzaghi & Peck, 1967].

Since 1969, remarkably few significant modifications to this original work have been adopted in practice. More recently, Sabatini et. al. [1999] proposed a more detailed design procedure based on the apparent earth pressure approach intended specifically for pre-tensioned, tied-back retaining walls in a comprehensive manual prepared for the US Department of Transportation, Federal Highway Administration. This manual is in wide use within the US and is gaining increasing acceptance within New Zealand and forms the basis for the worked example given below.

Little guidance is available for the design of tied-back retaining walls to resist seismic actions. Gravity retaining walls are normally designed using a pseudo-static approach: The active wedge of soil immediately behind the wall has an additional pseudo-static force component equal to the mass of soil within the wedge multiplied by acceleration. Typically, the resulting forces are resolved to derive a new critical wedge geometry and necessary wall pressure to achieve equilibrium, as in the Mononobe-Okabe (M-O) theory [Okabe, 1926; Mononobe and Matsuo, 1929].

Kramer [1996] has summarised the limited research available on the performance of tied-back walls during earthquakes. Very few reports of the behaviour of tied back walls during earthquakes are available. Ho et. al. [1990] surveyed ten anchored walls in the Los Angeles area following the Whittier earthquake of 1987 and concluded that they performed very well with little or no loss of integrity.

Sabatini et. al. [1999] recommends the use of the pseudo-static Mononobe-Okabe equations [Okabe, 1926; Mononobe and Matsuo, 1929] to calculate earthquake induced active earth pressures acting against the back face of a tied-back wall. A seismic coefficient from between one-half to two-thirds of the peak horizontal ground acceleration (0.5 PGA to 0.67 PGA) is recommended to provide a wall design that will limit deformations to small values acceptable for highway facilities. The length of the ground anchors may need to be
increased beyond that calculated for static design with the anchor bond zone located outside of the Mononobe-Okabe active wedge of soil. McManus [2008] provides a detailed design procedure for earthquake resistant design of tied-back retaining walls based on the recommendations of Sabatini et.al. Numerical analyses of several case studies showed that all of the walls designed using the procedure were robust and would be expected to perform very well, including those designed only to resist gravity loads. In some cases large permanent deformations were calculated (up to 400 mm) but these were for very large earthquakes (scaled peak ground acceleration of 0.6 g). In all cases the walls remained stable with anchor forces safely below ultimate tensile strength. Wall bending moments reached yield in some cases for the extreme earthquakes, but this is considered acceptable provided the wall elements are detailed for ductility.

Walls designed to resist low levels of horizontal acceleration (0.1 g and 0.2 g) showed significant improvements in performance over gravity only designs in terms of permanent displacement for relatively modest increases in cost. Walls designed to resist higher levels of horizontal acceleration (0.3 g and 0.4 g) showed additional improvements in performance but at much greater increases in cost.

The worked example given below uses the detailed procedure of Sabatini et. al [1999] “FHWA procedure” with modifications by McManus [2008] for the earthquake loading case. Increasingly, practitioners are relying on computer “black box” software to design tied-back walls with methodologies that range from fully elastic “beam-on-elastic-foundation” approaches to limiting equilibrium approaches. Caution is required when using “black box” software to ensure that all possible failure modes have been considered.

1.1 Possible modes of failure

Possible modes of failure for tied-back retaining walls are illustrated in cartoon fashion in Figure X.1. A complete design needs to address each of these modes of failure:

a) **Tensile failure of tendon**: The range of tendon loads must be established with suitable margins for safety.

b) **Grout/ground bond failure**: Generally this should always be established on site by proof testing given the difficulty in predicting the capacity and the dependence on installer skill and technique.

c) **Tendon/grout bond failure**: Prevented by reference to proven, commercial anchor details.

d) **Wall bending failure**: Actual wall bending moments are very difficult to predict because of the interaction between soil and structure stiffness and the non-linearity of soil stiffness. However, wall hinging does not necessarily create a mechanism provided the wall element is ductile.

e) **Passive failure at foot of wall**: Insufficient embedment depth for poles leads to passive failure of the soil immediately in front of the wall and instability of the wall and soil mass.

f) **Forward rotation of wall**: Staging of excavation is necessary to prevent forward rotation of wall prior to anchor installation. Wall needs sufficient bending strength
to resist cantilever moments for staged excavation. Anchors need to be of sufficient capacity and length to prevent forward rotation.

**g) Bearing failure underneath wall:** Caused by downwards component of anchor force. Check axial capacity of soldier piles, or, bearing capacity of foot of continuous wall. Bearing loads may be reduced by reducing the anchor inclination (15 degrees is a practical minimum).

**h) Failure by overturning:** Essentially same as (f). Anchors need to be of sufficient capacity and length to prevent forward rotation.

**i) Failure by sliding:** Possible mode for cohesionless soils. Factor of safety controlled by increasing depth of embedment of wall and/or poles. Factor of safety calculated using limiting equilibrium “wedge” analysis.

**j) Failure by rotation:** Possible mode for cohesive soils. Factor of safety controlled by increasing depth of embedment of wall and/or soldier piles. Factor of safety calculated using limiting equilibrium “Bishop” analysis or similar.
Figure X.1. Possible modes of failure for tied-back retaining walls
[Source: Sabatini et. al., 1999].

The following procedure addresses each of the above failure modes and is intended to be readily calculated by hand, although use of calculation software such as Mathcad or Excel will be useful for design iterations. The example calculations are made here using Mathcad.

1.2 Example Wall

The example wall is shown in Figure X.2 and consists of a 4 m high timber pole wall with ground anchor tie-backs. The wall is assumed to be protecting a levelled dwelling platform with a steep, 20 degree hill slope above. The wall is assumed to be located in the Christchurch Port Hills.

The following design assumptions were made:

**Soil type:** Port Hills loess

**Strength parameters:** $c = 0$, $\phi = 30$ degrees

(Drained strength parameters for Port Hills loess were assumed as recommended elsewhere in the Guidelines)

**Wall situation:** Case 4: Retaining wall protecting dwelling up-slope

**Seismic parameters:**

$C(T) = C_h(T)ZRN(T, D)$

Equation (1.1) from Guidelines

$C_h(T)$ 1.33 for Class C assuming shallow soil site

$Z = 0.3$ for Christchurch for ULS
R = Return period factor = 1.0 for Importance Level 2 walls, ULS
N(T,D) = Near fault factor which may be taken = 1.0 for residential retaining walls

\[ C(T) = 0.3 \times 1.33 = 0.4 \]

\[ C(T, Atopo) = C(T)Atopo \] Equation (1.2) from Guidelines

\[ Atopo = 1.0 \text{ assuming site is not near cliff edge or ridge top} \]

\[ C(T, Atopo) = 0.4 \times 1.0 = 0.4 \]

\[ kh = C(T, Atopo)W_d \] Equation (1.3) from Guidelines

\[ W_d = \text{wall displacement factor, given in Table 2 from Guidelines as 0.4} \]

\[ kh = 0.4 \times 0.4 = 0.16 \]

Note that by adopting \( W_d = 0.4 \) it is implicitly assumed that the wall and the retained ground are likely to yield and accumulate permanent displacement as a result of the design earthquake. Wall elements including the poles and anchor tendons must be sufficiently robust and ductile to accommodate the displacement.

Figure X.2. Tied-back retaining wall example.

The basic dimensions shown in Figure X.2 were developed as follows:
Distance of anchor from top of wall = 1.2 m (from trial and error during calculations, see below)

Anchor inclination 15 degrees minimum to permit efficient grouting, 20 degrees required in this case to achieve recommended cover depth of soil over anchor bond zone

Anchor free length. Minimum = 3 m for bar anchor, 4 m for strand anchor, must extend beyond the failure plane for the active soil wedge (which will be different for the gravity and earthquake cases). For this example, anchor free length = 4 m (see Figure X.2.)

Anchor bond length: To be determined.

Step 1. Initial trial geometry

The depth of excavation and depth to each row of anchors needs to be estimated as a first step, based on experience or trial and error. Typically from 1m to 2 m.

Step 2. Prepare apparent earth pressure diagram (gravity case)

The total load acting against the wall from earth pressure for gravity only is based on the earth pressure envelopes recommended by Terzaghi & Peck [1967] (Figure X.3) and modified by Sabatini et. al. [1999] for tied-back walls (Figure X.4). For the earthquake load case, $K_A$ should be calculated using the M-O equations as $K_{AEH}$. The interface friction angle between the back of the wall and the soil should be conservatively assumed = 0, because the active soil wedge and wall may both move downwards together (i.e. without any vertical component of friction).

![Figure X.3. Apparent earth pressure envelope for sand for braced excavation](Terzaghi & Peck, 1967)
Figure X.4. Apparent earth pressure diagram for tied-back walls with one level of ground anchors in sand. [Source: Sabatini et. al., 1999]

FHWA procedure for single anchor wall in sand

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{wall}$</td>
<td>4 m</td>
<td>Height of wall</td>
</tr>
<tr>
<td>$H_1$</td>
<td>1.2 m</td>
<td>Distance to anchor from top of wall</td>
</tr>
<tr>
<td>$P_v$</td>
<td>$0 \cdot \frac{KN}{m^2}$</td>
<td>Surcharge on active wedge (factored)</td>
</tr>
<tr>
<td>$K_{ah}$</td>
<td>0.44</td>
<td>$K_a$, horiz. compt., $\phi = 30$ deg, $\delta = 0$, $i = 20$ deg.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$18 \cdot \frac{KN}{m^3}$</td>
<td>Unit weight of retained soil</td>
</tr>
<tr>
<td>$L_s$</td>
<td>1.5 m</td>
<td>Pole spacing</td>
</tr>
</tbody>
</table>

Apparent earth pressure, $p$

$$p = K_{ah} \gamma H_{wall}$$

$$p = 31.68 \cdot \frac{KN}{m^2}$$
Step 3. Calculate anchor design load and reaction force required at base of wall (gravity case)

Moment about base gives anchor force, \( T \) and base reaction, \( R \)

\[
I_a = \frac{H_1}{3} \quad I_a = 0.8 \text{ m}
\]
\[
I_b = \frac{H_{wall}}{3} \quad I_b = 1.333 \text{ m}
\]
\[
I_c = \frac{H_{wall} - H_1}{3} \quad I_c = 1.867 \text{ m}
\]
\[
M_{base} = P \cdot L_s \left[ \frac{I_c}{3} + I_b \left( \frac{L_b}{2} + L_c \right) + \frac{I_a}{2} \left( \frac{L_a}{3} + L_b + L_c \right) \right]
\]
\[
T_{anchor} = \frac{M_{base}}{H_{wall} - H_1} \quad T_{anchor} = 100.571 \text{ KN}
\]
\[
R_{base} = \frac{2}{3} P \cdot H_{wall} \cdot L_s - T_{anchor} \quad R_{base} = 26.149 \text{ KN}
\]

Step 4. Calculate pole bending moments (gravity case)

Cantilever pole bending moment above

\[
M_c = L_s \cdot P \left[ \frac{L_a}{3} \left( \frac{L_a}{3} + \frac{H_1}{3} \right) + \frac{H_1^2}{18} \right] \quad M_c = 16.474 \text{ KN-m}
\]

Maximum pole bending moment below anchor at location of zero shear

\[
V_{La} = L_s \cdot P \cdot \frac{L_a}{2} \quad \text{SF at distance } L_a \text{ from top of wall}
\]
\[
V_{H1} = V_{La} + L_s \cdot P \left( H_1 - L_a \right) \quad \text{SF at distance } H_1 \text{ from top of wall}
\]
\[
Z_{vzero} = \frac{T_{anchor} - V_{H1}}{L_s \cdot P} + L_a \quad Z_{vzero} = 2.116 \text{ m} \quad \text{distance from top of wall to zero shear assuming less than } L_a + L_b
\]
\[
L_a + L_b = 2.133 \text{ m} \quad \text{OK}
\]
\[
M_{max} = \frac{L_s \cdot P \cdot L_a}{2} \left( Z_{vzero} - \frac{L_a}{3} \right) + \frac{L_s \cdot P \cdot (Z_{vzero} - L_a)^2}{2} - T_{anchor} \left( Z_{vzero} - H_1 \right)
\]
\[
M_{max} = -20.899 \text{ KN-m} \quad \text{Optimise pole bending moments by varying } H_1
\]
Step 5. Determine depth of pole embedment (gravity case)

Calculate required depth of embedment for soldier piles to resist wall base reaction using Broms [1965] or similar. If the pole spacing is less than three diameters, then treat as a continuous wall. For the worked example, it was assumed that the poles will be set into 500 mm diameter holes and concrete filled. Therefore, the following calculations treat the embedded poles as a continuous palisade.

(Note that in this case the internal stability check below was found to govern the depth of embedment).

Step 6. Check internal stability of the wall (gravity case)

A possible internal failure mechanism is shown in Figure X.5 with an active failure wedge immediately behind the wall, a passive wedge immediately in front of the embedded toe of the wall, and the anchor(s) developing their proven test capacity (normally 1.33 times the design load or 80 percent of the anchor tensile capacity).

The factor of safety should be $FS > 1.5$ for the gravity case.
Figure X.5. Internal and external failure mechanisms for tied back walls.  
[Source: Sabatini et.al 1999]

**Internal stability check**

\[ F_H = \frac{T_{anchor}}{L_s} \]  
Anchor horizontal force (proven test capacity)

\[ P_{ah} = K_{ah} \left[ 0.5 \cdot \gamma \cdot H_{wall} + \gamma \cdot H_{wall} \cdot D + 0.5 \cdot \gamma_{base} \cdot D^2 + p_v \cdot (H_{wall} + D) \right] \]

\[ P_{ph} = 0.5 \cdot \gamma_{base} \cdot K_{ph} \cdot D^2 \]

\[ H_{net} = P_{ah} - P_{ph} - F_H \]

\[ H_{net} = -54.67 \text{ KN/m} < 0 \text{ for stability} \]

\[ F_S = \frac{P_{ph} + F_H}{P_{ah}} \]

\[ F_S = 1.511 \]

> 1.5 for gravity case  
(adjust D as required)
Step 7. Prepare apparent earth pressure diagram (earthquake case)

FHWA procedure for single anchor wall in sand

\[
\begin{align*}
H_{\text{wall}} & := 4 \text{-m} & \text{Height of wall} \\
H_1 & := 1.2 \text{-m} & \text{Distance to anchor from top of wall} \\
\rho_{\text{v}} & := 0.6 \text{-kN/m}^2 & \text{Surcharge on active wedge} \\
K_{\text{ach}} & := 0.75 & \text{K_a, M-O eqns, horiz. comp., } \phi = 30 \text{ deg, } \theta = 0, i = 20 \text{ deg, } k_h = 0.16. \\
\gamma & := 18.3 \text{-kN/m}^3 & \text{Unit weight of retained soil} \\
L_s & := 1.5 \text{-m} & \text{Pole spacing}
\end{align*}
\]

Apparent earth pressure, \( p \)

\[
p := K_{\text{ach}} \gamma H_{\text{wall}}
= 54 \text{-kN/m}^2
\]

Step 8. Calculate anchor design load and reaction force required at base of wall (earthquake case)

Moment about base gives anchor force, \( T \) and base reaction, \( R \)

\[
\begin{align*}
I_a & := \frac{2}{3} H_1 & I_a = 0.8 \text{-m} \\
I_b & := \frac{H_{\text{wall}}}{3} & I_b = 1.333 \text{-m} \\
I_c & := \frac{H_{\text{wall}} - H_1}{3} & I_c = 1.367 \text{-m} \\
M_{\text{base}} & := p I_s \left[ \frac{I_c^2}{3} + I_b \left( \frac{I_b}{2} + I_c \right) + \frac{I_a}{2} \left( \frac{I_a}{3} + I_b + I_c \right) \right]
\end{align*}
\]

\[
\begin{align*}
T_{\text{anchor}} & := \frac{M_{\text{base}}}{H_{\text{wall}} - H_1} & T_{\text{anchor}} = 171.429 \text{-KN} \\
R_{\text{base}} & := \frac{2}{3} p H_{\text{wall}} I_s - T_{\text{anchor}} & R_{\text{base}} = 44.571 \text{-KN}
\end{align*}
\]

Note that the earthquake case gives a much greater anchor design load than the gravity case, cf. 100 KN from Step 3.
Step 9. Calculate pole bending moments (earthquake case)

Cantilever pole bending moment above anchor

\[ M_c = L_s P \left[ \frac{L_a}{2} \left( \frac{L_a}{3} + \frac{H_1}{3} \right) + \frac{H_1^2}{18} \right] \]

\[ M_c = 28.08 \text{ kN-m} \]

Maximum pole bending moment below anchor at location of zero shear

\[ V_{La} = L_s P \frac{L_a}{2} \]

SF at distance \( L_a \) from top of wall

\[ V_{H1} = V_{La} + L_s P \left( H_1 - L_a \right) \]

SF at distance \( H_1 \) from top of wall

\[ Z_{vzero} = \frac{T_{anchor} - V_{H1}}{L_s P} + L_a \]

\[ Z_{vzero} = 2.116 \text{ m} \]

Distance from top of wall to zero shear assuming less than \( L_a + L_b \)

\[ L_a + L_b = 2.133 \text{ m} \]

OK

\[ M_{max} = \frac{L_s P \cdot L_a}{2} \left( Z_{vzero} - \frac{2}{3} L_a \right) + \frac{L_s P \cdot (Z_{vzero} - L_a)^2}{2} - T_{anchor} \cdot (Z_{vzero} - H_1) \]

\[ M_{max} = -35.623 \text{ kN-m} \]

Optimise pole bending moments by varying \( H_1 \)

Pole bending design

\[ \psi = 1.0 \]

Load factor = 1.0 for earth pressure for EQ case

\[ M_{star} = M_{max} \psi \]

\[ M_{star} = -35.623 \text{ kN-m} \]

OR

\[ M_{star2} = M_c \psi \]

\[ M_{star2} = 28.08 \text{ kN-m} \]

AND

\[ V_{star} = V_{H1} \psi \]

\[ V_{star} = 64.8 \text{ kN} \]

Shear above anchor

OR

\[ V_{star2} = \left( T_{anchor} - V_{H1} \right) \psi \]

\[ V_{star2} = 106.629 \text{ kN} \]

Shear below anchor

(Note that the critical factored bending moment in this case is similar to the gravity case in Step 4, cf. -31 KNm. Also, the load duration factor for timber (k1) is 1.0 for seismic and 0.6 for long-term/permanent loading making the gravity case more critical for the example. However, both load-cases should be checked separately.)

Step 10. Determine depth of embedment (earthquake case)

Check that the depth of embedment of the poles is adequate for the earthquake case by considering a simplified sliding analysis as shown in Figure X.6. For the earthquake case, the undrained shear strength of the foundation soil may be assumed for Port Hills loess when calculating the passive soil resistance, \( S_u = 50 \text{ KN/m}^2 \) was assumed for the example. The passive soil distribution is as shown in Figure X.6 with the cohesive contribution = 2 \( c \), \( c = S_u \), and \( K_p = 1 \) for \( \phi = 0 \). Where the ground surface immediately in front of the wall is exposed, the cohesive component of passive resistance may be ineffective near to the ground surface.
because of desiccation and cracking. For the example, the upper 0.5 m of cohesive passive resistance was neglected. For other situations where the ground surface is protected by pavement it may be appropriate to include the passive soil resistance over the full depth of embedment, using judgement.

Figure X.6. Analytical model for internal stability check (earthquake case).

(Note that in this case the internal stability check below was found to govern the depth of embedment).

Step 11. Check internal stability of the wall (earthquake case)

The factor of safety for the internal failure mechanism shown in Figure X.5 is checked again for the earthquake case. Displacement of the wall during peak accelerations pulses may stretch the tendon and increase the amount of pre-load in the tendon. Rupture of the
tendon is unlikely provided there is sufficient un-bonded free-length and the tendon material is suitably ductile (e.g. Macalloy 1030 bar is rated at 6% minimum elongation, equivalent to 240 mm for the design example with a free length of 4 m)

The factor of safety should be $FS > 1.0$ for the earthquake case.

**Step 12. Selection of anchor.**
The earthquake case was found to govern the calculation of the design load for the anchor, with a required horizontal load of 171 KN @ 1.5 m spacing. A more efficient design might be to provide anchors at 3 m centres with short waler beams to spread the load between pairs of poles. For example:

- Anchor spacing = 3 m
- Anchor inclination angle = 20 degrees:
- Anchor design load = 342 KN / Cos 20 = 364 KN
- Anchor test load = 364 x 1.33 = 484 KN
- Anchor minimum characteristic tensile strength = 484/0.8 = 605 KN (i.e. maximum test load = 0.8 x anchor characteristic tensile strength)

Refer to FHWA guidelines for more advice or BS 8081: 1989

**Step 13. External stability check.**
The external stability case (refer to Figure D5) is controlled by the total length of the ground anchor and should be checked once the anchor length has been determined. A wedge analysis may be undertaken using hand calculations or proprietary slope stability software used.

For the worked example, analysis using PLAXIS indicated that a minimum anchor length of 9.5 m would be required to achieve the desired $FS ≥ 1$ for the design acceleration $k_h = 0.16$. 
Other issues.

The global stability of the wall and surrounds may need to be checked in certain cases, for example where the ground in front of the retaining wall is sloping away. Also where there is weak ground below or in front of the toe of the wall.

The axial capacity of the poles acting as piles may need to be checked in some cases, for example where the anchor forces are very high or steeply inclined, and where the ground below the pole foundations is weak.