

## Worked Example 3 (Version 1)

### Design of concrete crib retaining walls to resist earthquake loading for residential sites

Worked example to accompany MBIE Guidance on the seismic design of retaining structures for residential sites in Greater Christchurch (Version 2) November 2014

#### Introduction

Concrete crib and timber crib retaining walls are a type of gravity wall which comprises a system of interlocking header and stretcher blocks to retain granular fill that provides the necessary stabilising mass to the wall. Crib walls are commonly used for residential purposes such as stabilising building platforms and driveway access. They are very adaptable and can be straight, curved, or angled and incorporate landscape features if required. Heights typically vary from 2 m to 12 m. Crib walls are able to sustain differential settlement. They have been proven over many decades of use in New Zealand.

There was quite a wide variation in seismic performance observed for crib walls during the Canterbury earthquake sequence. It appears that this variability is much more strongly influenced by construction details and practice rather than fundamental design. A particular construction issue was the use of rounded gravel fill within the wall units. Rounded material tends to shake out leaving voids between the block units and settlement of the ground or pavements above the wall. Angular, crushed rock filling should always be used and separated from finer grained soils by good quality filter fabric.

The following worked example is for a typical concrete crib retaining wall supporting a cut slope face on the up-slope side of a dwelling. The design analysis is based on a conventional gravity wall analysis in which the wall and soil encapsulated by the crib units is assumed to act as a rigid block.

#### 1.1 Possible modes of failure

Possible modes of failure for crib retaining walls are illustrated in cartoon fashion in Figure X.1. A complete design should address each of these modes of failure where appropriate.

- a) **Foundation bearing failure:** A bearing failure of the soil under the toe of the foundation and a forwards rotation of the wall. Crib walls should be built on concrete pad foundations at least as wide as the crib units. Crib walls are usually constructed on a 4V:1H batter that greatly improves the overall stability of the wall and reduces the eccentricity of loading on the foundation pad.
- b) **Internal shear failure of wall:** The design of the interlocking crib units is intended to provide a high resistance to internal shear failure, together with the use of good quality angular, crushed rock filling.
- c) **Crushing failure of crib units:** Crushing of crib units is possible under high overturning loads.
- d) **Sliding failure of wall:** Possible mode for non-cohesive soils. Wall moves outwards with passive failure of soil in front of foundation and active failure of soil behind

wall. If necessary, a key can be added beneath the foundation to improve sliding resistance.

- e) **Deep seated rotational failure:** Possible mode for cohesive soils. Factor of safety calculated using limiting equilibrium “Bishop” analysis or similar. Unlikely to govern design unless wall is embedded into sloping ground with sloping backfill or there is a weak layer at the toe of the wall.

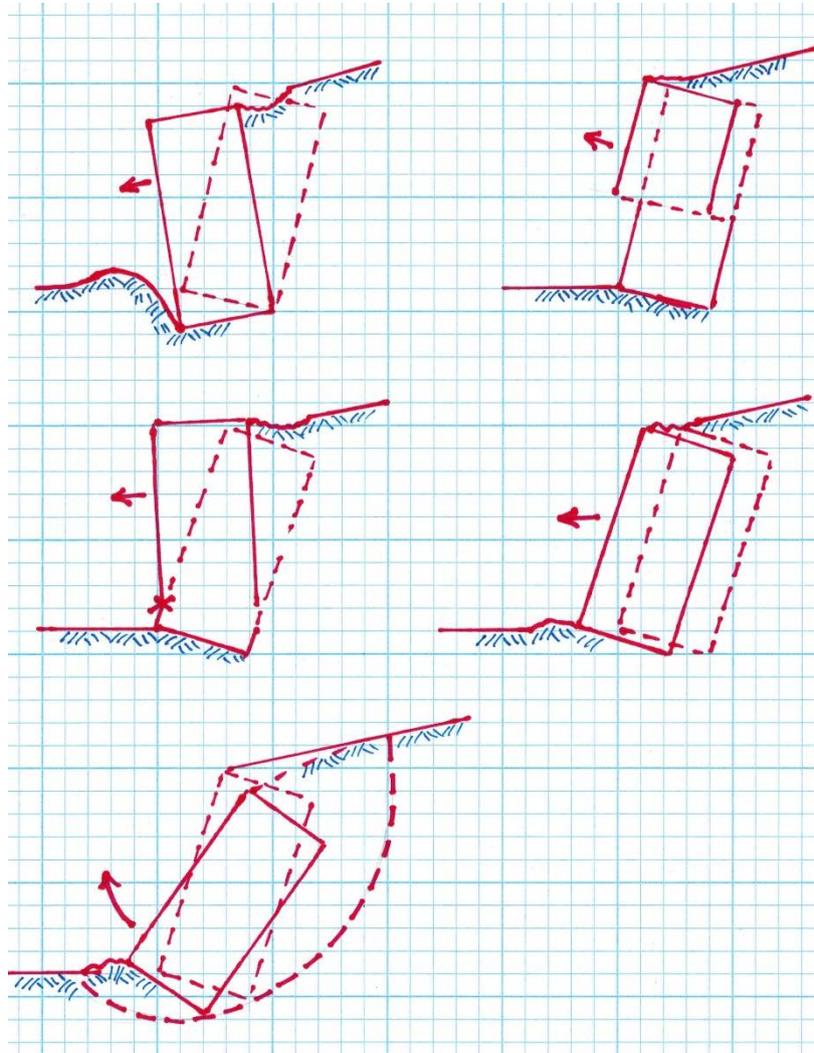


Figure X.1. Possible modes of failure for crib retaining walls.

The following worked example uses a simplified LRFD design procedure with load and resistance factors taken from B1/VM4. The procedure is considered suitable for common residential situations with competent soils.

This procedure is intended to be readily calculated by hand, although use of calculation software such as Mathcad or Excel will be useful for design iterations. The example calculations are made here using Mathcad.

## 1.2 Example Wall

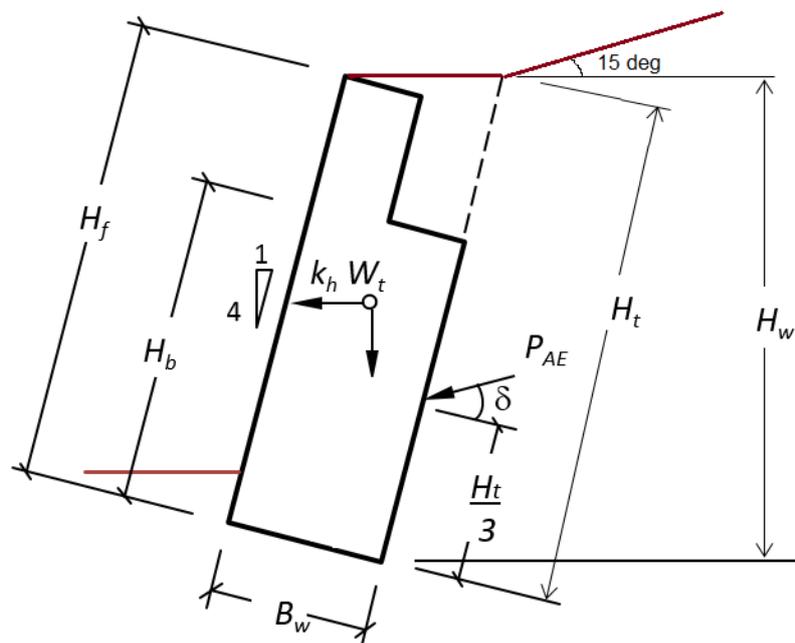


Figure X.2. Concrete crib wall example.

The analysis definition for a double-tier type crib wall is shown in Figure X.2. The wall was assumed to be constructed on a concrete strip footing of the same width as the wall. The wall is assumed to be located in the Christchurch Port Hills and supporting a cut batter on the up-slope side of a dwelling (Case 4 in the Guidelines).

The following design assumptions were made:

Soil type: Port Hills loess

Strength parameters:  $c = 0$ ,  $\phi = 30$  degrees

Drained strength parameters for Port Hills loess were assumed for the long term, gravity only load case. For the earthquake load case, the foundations in loess were designed assuming undrained strength,  $c = 50 \text{ kN/m}^2$ ,  $\phi = 0$  degrees. (Following the recommendations given in the Guidelines).

Wall situation: Case 4: Retaining wall protecting dwelling up-slope

Surcharge: No surcharge is assumed

Back-slope: A back-slope angle of 15 degrees is assumed

Seismic parameters:

$C(T) = C_h(T)ZRN(T, D)$  Equation (1.1) from Guidelines

$C_h(T)$  1.33 for Class C assuming shallow soil site

$Z = 0.3$  for Christchurch for ULS

$R =$  Return period factor = 1.0 for Importance Level 2 walls, ULS

$N(T, D) =$  Near fault factor which may be taken = 1.0 for residential retaining walls

$C(T) = 0.3 \times 1.33 = 0.4$

$$C(T, A_{topo}) = C(T)A_{topo} \quad \text{Equation (1.2) from Guidelines}$$

$$A_{topo} = 1.0 \text{ assuming site is not near cliff edge or ridge top}$$

$$C(T, A_{topo}) = 0.4 \times 1.0 = 0.4$$

$$k_h = C(T, A_{topo})W_d \quad \text{Equation (1.3) from Guidelines}$$

$W_d = 0.4$  wall displacement factor, given in Table 2 from Guidelines (refer to Table 1 for wall case, then Table 2 for  $W_d$ )

$$k_h = 0.4 \times 0.4 = 0.16$$

Note that by adopting  $W_d = 0.4$  it is implicitly assumed that the wall and the retained ground are likely to yield and accumulate permanent displacement during the design earthquake. Wall elements must be sufficiently resilient and/or ductile to accommodate the displacement. Some settlement of retained material behind the wall should also be expected following an earthquake.

### **Step 1. Initial trial geometry**

The main variables for geometry are defined in Figure X.2 with the trial values given below:

$H_F := 4.5\text{-m}$	Slope height of wall
$B_w := 2.2\text{-m}$	Width of footing
$\eta := \text{atan}\left(\frac{1}{4}\right)$	Wall slope angle
$i := 15\text{-deg}$	Backfill slope angle
$\phi := 30\text{-deg}$	Soil friction angle
$\delta_a := 0.67 \cdot \phi$	Interface friction angle, wall virtual back face
$\gamma := 18 \cdot \frac{\text{KN}}{\text{m}^3}$	Soil unit weight
$\gamma_{\text{wall}} := 18 \cdot \frac{\text{KN}}{\text{m}^3}$	Average unit weight of wall units and backfill

### **Step 2. Foundation bearing (gravity case)**

The foundation bearing capacity (gravity case) will usually govern the design of the wall dimensions and is checked first. The soil under the toe of the foundation in particular is working very hard to resist the vertical bearing loads, sliding shear, and to provide passive resistance to sliding.

For the example, the footing is assumed to be 200 mm thick and embedded flush with the ground surface.

For the following simplified procedure, the “middle third rule” is applied, whereby the wall foundation is dimensioned so that the resultant force acts through the “middle third” of the footing. If the “middle third rule” is not applied, then a more rigorous analysis of the bearing capacity of the wall foundation should be undertaken (e.g. Pender, 2015 – in press).

The foundation of the crib wall is tilted at an angle of 1V:4H and so it is convenient to resolve all forces acting on the wall to components acting either perpendicular to the back face of the wall or parallel to the wall instead of vertical and horizontal.

The bearing capacity of the foundation **must** be calculated taking into account the effect of simultaneous horizontal loads applied to the foundation from the soil pressure (i.e. by applying load inclination factors), and using the reduced, effective width of the foundation from the eccentricity of the resultant vertical load. Where there is confidence in the properties of the soil backfill in front of the toe of the footing, then the net horizontal load considered when calculating the load inclination factors for the bearing capacity may be reduced by the passive soil force acting against the footing (refer to Brinch-Hansen 1970), in which case the depth factors must be set to 1.0 (i.e. the shear strength of the soil above the founding depth of the footing cannot be counted twice). In the worked example, the passive soil resistance has been neglected (conservatively) when calculating the load inclination factors and bearing capacity.

The bearing capacity of the foundation is also affected by the *tilt* of the footing and so tilt factors are applied (see detailed bearing capacity calculations appended to the example):

#### LRFD parameters

$\Phi_{bc} := 0.5$	Resistance factor for bearing capacity, gravity case
$\Phi_{sl} := 0.8$	Resistance factor for sliding, gravity case
$\Phi_p := 0.5$	Resistance factor for passive earth pressure, gravity case
$\alpha_{G\_stab} := 0.9$	Load factor for self-weight (stabilising)
$\alpha_{G\_destab} := 1.2$	Load factor for self-weight (de-stabilising)
$\alpha_{EP\_static} := 1.5$	Load factor for earth pressure, gravity case (de-stabilising)

#### Computed parameters

$H_s := B_w \cdot \tan(\eta)$	$H_s = 0.55 \text{ m}$	
$H_t := H_f + H_s$	$H_t = 5.05 \text{ m}$	Length of wall backface
$H_w := H_t \cdot \cos(\eta)$	$H_w = 4.899 \text{ m}$	Height of wall (projected)
$W_1 := H_f \cdot B_w \cdot \gamma_{wall}$	$W_1 = 178.2 \cdot \frac{\text{KN}}{\text{m}}$	Weight, virtual wall
$W_2 := 0.5 \cdot B_w \cdot H_s \cdot \gamma$	$W_2 = 10.89 \cdot \frac{\text{KN}}{\text{m}}$	Weight, soil wedge
$K_a := 0.253$	From M-O eqns. $k_h=0$ , $\phi = 30 \text{ deg}$ , $\delta = 2\phi/3$ , $i = 15 \text{ deg}$ , $\beta = -14 \text{ deg}$	

**Check "middle third rule"**

Factored moments about toe, divided by factored perpendicular forces neglecting passive resistance, which may not be mobilised.

$$P_a := 0.5 \cdot K_a \cdot \gamma \cdot H_w^2 \quad P_a = 54.653 \cdot \frac{\text{KN}}{\text{m}} \quad \text{Active thrust, soil weight}$$

$$P_{aT} := P_a \cdot \cos(\delta_a) \quad P_{aT} = 51.325 \cdot \frac{\text{KN}}{\text{m}} \quad \text{Perpendicular component}$$

$$P_{aL} := P_a \cdot \sin(\delta_a) \quad P_{aL} = 18.782 \cdot \frac{\text{KN}}{\text{m}} \quad \text{Parallel component}$$

$$M_{aT} := \left( P_{aT} \cdot \frac{H_t}{3} \right) \cdot \alpha_{EP\_static} \quad M_{aT} = 129.595 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}} \quad \text{Moment, perpendicular active pressure (+ve)}$$

$$M_{aL} := P_{aL} \cdot B_w \quad M_{aL} = 41.321 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}} \quad \text{Moment, parallel active pressure (-ve)}$$

$$M_{G1} := W_1 \left( \cos(\eta) \cdot \frac{B_w}{2} + \sin(\eta) \cdot \frac{H_f}{2} \right) \cdot \alpha_{G\_stab} \quad M_{G1} = 258.671 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}}$$

$$M_{G2} := W_2 \left[ \cos(\eta) \cdot \frac{2 \cdot B_w}{3} + \sin(\eta) \cdot \left( H_f + \frac{H_s}{3} \right) \right] \cdot \alpha_{G\_stab} \quad M_{G2} = 25.078 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}}$$

$M_{G1}, M_{G2}$  = Restoring moments from self weight of wall and soil above (-ve)

(See Figure X.3 for definition of  $W_1$  and  $W_2$ )

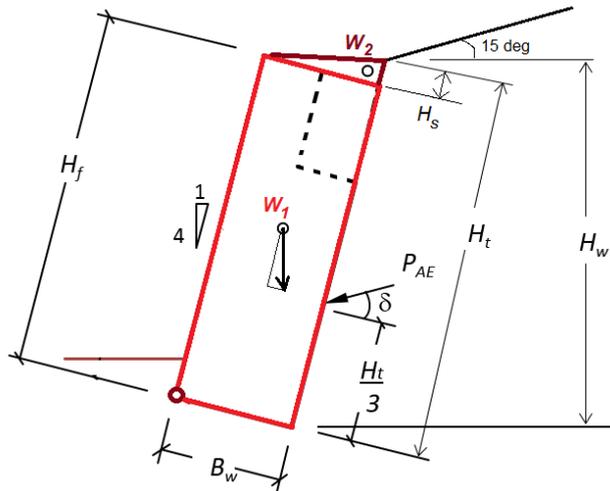


Figure X3. Virtual wall used for analysis, showing  $W_1$  and  $W_2$ .

$$M_{net} := M_{aT} - M_{aL} - M_{G1} - M_{G2}$$

$$M_{net} = -195.475 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}} \quad \text{Net moment must be } < 0 \text{ for stability}$$

$$P_T := (W_1 + W_2) \cdot \cos(\eta) \cdot \alpha_{G\_stab} + P_{aL} \quad \text{Factored net perpendicular load on footing}$$

$$P_T = 183.882 \cdot \frac{\text{KN}}{\text{m}} \quad \text{Net moment about toe, neglecting passive thrust}$$

$$L_{net} := \frac{-M_{net}}{P_T} \quad L_{net} = 1.063 \text{ m} \quad \text{Line of action of net vertical force (distance from toe)}$$

$$L_{third} := \frac{1}{3} \cdot B_w \quad L_{third} = 0.733 \text{ m} \quad 2 \cdot L_{third} = 1.467 \text{ m}$$

Adjust wall proportions until line of action is within "middle third"

Note that the parallel component (i.e. parallel to the back face of the wall) of active thrust is not factored (i.e.  $\alpha = 1$ ). The perpendicular component of active thrust is factored ( $\alpha = 1.5$ )

to account for uncertainty of soil properties. But, uncertainty in soil properties does not significantly affect the parallel component which will remain about the same even if the actual soil friction angle is less than assumed.

The self-weight components are here factored **down** ( $\alpha = 0.9$ ) to account for uncertainty because they are “stabilising” in this context, even though contributing to the load on the footing.

**Check bearing capacity**

The “effective” width of the footing must be established, together with the net perpendicular and parallel loads acting on the footing:

$$B_{\text{eff}} := 2 \cdot L_{\text{net}} \quad B_{\text{eff}} = 2.126 \text{ m} \quad \text{Effective footing width (not wider than footing)}$$

$$V_u := P_T \quad V_u = 183.882 \cdot \frac{\text{KN}}{\text{m}} \quad \text{Ultimate perpendicular load on footing}$$

$$H_u := P_{aT} \cdot \alpha_{EP\_static} - (W_1 + W_2) \cdot \sin(\eta) \cdot \alpha_{G\_stab} \quad \text{Ultimate parallel load on footing}$$

$$H_u = 35.712 \cdot \frac{\text{KN}}{\text{m}}$$

Detailed bearing capacity calculations are appended, and give the following result:

$$q_u = 197.333 \cdot \frac{\text{KN}}{\text{m}^2} \quad \text{Ultimate bearing pressure}$$

$$V_{\text{star}} := q_u \cdot B_{\text{eff}} \cdot \Phi_{bc} \quad V_{\text{star}} = 209.774 \cdot \frac{\text{KN}}{\text{m}} \quad V_u = 183.882 \cdot \frac{\text{KN}}{\text{m}}$$

$V_{\text{star}} > V_u$  therefore bearing capacity OK for gravity case.

**Step 3. Wall sliding on base (gravity case)**

The sliding analysis is carried out with reference to the model shown in Figure X.3. All of the self-weight components are here factored **down** ( $\alpha = 0.9$ ) to account for uncertainty because they are “stabilising” in this context.

The passive resistance of the soil acting against the front of the foundation pad is neglected as being comparatively small in this example.

**Check wall sliding on base**

$$H_{\text{star}} := V_u \cdot \tan(\phi) \cdot \Phi_{s1} \quad \text{Factored ultimate resistance (friction under footing)}$$

$$H_{\text{star}} = 84.931 \cdot \frac{\text{KN}}{\text{m}} \quad H_u = 35.712 \cdot \frac{\text{KN}}{\text{m}}$$

Factored resistance > factored load therefore OK.

**Step 4. Foundation bearing (earthquake case)**

The foundation bearing capacity is checked for the earthquake case using the same geometry developed for the gravity case and including the earthquake inertia loads from the self-weight of the wall according to the analytical model shown in Figure X.3.

For the earthquake case, the undrained shear strength of the foundation soil may be assumed for Port Hills loess. For the example,  $S_u = 50 \text{ KN/m}^2$  was assumed.

Using the same simplified procedure as for the gravity case, the “middle third rule” is again checked.

For the earthquake case, the LRFD parameters are all set to unity, as discussed in the guidelines, assuming that the loess foundation soil will not be subject to strength loss during earthquake shaking or strain softening as a result of soil yielding.

$$K_{aE} := 0.439 \quad \text{From M-O eqns. } k_h=0.16, \phi = 30 \text{ deg}, \delta = 2\phi/3, i = 15 \text{ deg}, \beta = -14 \text{ deg}$$

**Check "middle third rule"**

Factored moments about toe, divided by factored perpendicular forces neglecting passive resistance, which may not be mobilised.

$P_a := 0.5 \cdot K_{aE} \cdot \gamma \cdot H_w^2$	$P_a = 94.833 \cdot \frac{\text{KN}}{\text{m}}$	Active thrust, soil weight
$P_{aT} := P_a \cdot \cos(\delta_a)$	$P_{aT} = 89.057 \cdot \frac{\text{KN}}{\text{m}}$	Perpendicular component
$P_{aL} := P_a \cdot \sin(\delta_a)$	$P_{aL} = 32.59 \cdot \frac{\text{KN}}{\text{m}}$	Parallel component
$I_{1T} := k_h \cdot W_1 \cdot \cos(\eta)$	$I_{1T} = 27.661 \cdot \frac{\text{KN}}{\text{m}}$	Inertia of wall , perpendicular compt.
$I_{2T} := k_h \cdot W_2 \cdot \cos(\eta)$	$I_{2T} = 1.69 \cdot \frac{\text{KN}}{\text{m}}$	Inertia of soil wedge , perpendicular compt.
$I_{1L} := k_h \cdot W_1 \cdot \sin(\eta)$	$I_{1L} = 6.915 \cdot \frac{\text{KN}}{\text{m}}$	Inertia of wall , parallel compt.
$I_{2L} := k_h \cdot W_2 \cdot \sin(\eta)$	$I_{2L} = 0.423 \cdot \frac{\text{KN}}{\text{m}}$	Inertia of soil wedge , parallel compt.
$M_{aT} := \left( P_{aT} \cdot \frac{H_t}{3} \right)$	$M_{aT} = 149.913 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}}$	Moment, perpendicular active pressure (+ve)
$M_{aL} := P_{aL} \cdot B_w$	$M_{aL} = 71.699 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}}$	Moment, parallel active pressure (-ve)
$M_{I1} := I_{1T} \cdot \frac{H_f}{2} - I_{1L} \cdot \frac{B_w}{2}$	$M_{I1} = 54.63 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}}$	Moment, inertia of wall
$M_{I2} := I_{2T} \cdot \left( H_f + \frac{H_s}{3} \right) - I_{2L} \cdot \frac{2 \cdot B_w}{3}$	$M_{I2} = 7.297 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}}$	Moment, inertia of wall
$M_{G1} := W_1 \cdot \left( \cos(\eta) \cdot \frac{B_w}{2} + \sin(\eta) \cdot \frac{H_f}{2} \right)$	$M_{G1} = 287.412 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}}$	
$M_{G2} := W_2 \cdot \left[ \cos(\eta) \cdot \frac{2 \cdot B_w}{3} + \sin(\eta) \cdot \left( H_f + \frac{H_s}{3} \right) \right]$	$M_{G2} = 27.865 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}}$	

$M_{G1}, M_{G2}$  = Restoring moments from self weight of wall and soil above (-ve)

$$M_{\text{net}} := M_{aT} + M_{I1} + M_{I2} - M_{aL} - M_{G1} - M_{G2}$$

$$M_{\text{net}} = -175.136 \cdot \frac{\text{KN}\cdot\text{m}}{\text{m}} \quad \text{Net moment must be } < 0 \text{ for stability}$$

$$V_u := (W_1 + W_2) \cdot \cos(\eta) + P_{aL} + I_{1L} + I_{2L} \quad \begin{array}{l} \text{Factored net perpendicular load on footing} \\ \text{Net moment about toe, neglecting passive thrust} \end{array}$$

$$V_u = 223.372 \cdot \frac{\text{KN}}{\text{m}}$$

$$L_{\text{net}} := \frac{-M_{\text{net}}}{V_u} \quad L_{\text{net}} = 0.784 \text{ m} \quad \text{Line of action of net vertical force (distance from toe)}$$

$$L_{\text{third}} := \frac{1}{3} \cdot B_w \quad L_{\text{third}} = 0.733 \text{ m} \quad 2 \cdot L_{\text{third}} = 1.467 \text{ m}$$

Adjust wall proportions until line of action is within "middle third"

The line of action of the force perpendicular to the wall footing is still within the "middle third".

#### Check bearing capacity

The "effective" width of the footing must be established, together with the net perpendicular and parallel loads acting on the footing:

$$B_{\text{eff}} := 2 \cdot L_{\text{net}} \quad B_{\text{eff}} = 1.568 \text{ m} \quad \text{Effective footing width (not wider than footing)}$$

$$V_u = 223.372 \cdot \frac{\text{KN}}{\text{m}} \quad \text{Ultimate perpendicular load on footing}$$

$$H_u := P_{aT} + I_{1T} + I_{2T} - (W_1 + W_2) \cdot \sin(\eta) \quad \text{Ultimate parallel load on footing}$$

$$H_u = 72.547 \cdot \frac{\text{KN}}{\text{m}}$$

Detailed bearing capacity calculations are appended, and give the following result:

$$q_u = 164.032 \cdot \frac{\text{KN}}{\text{m}^2} \quad \text{Ultimate bearing pressure}$$

$$V_{\text{star}} := q_u \cdot B_{\text{eff}} \quad V_{\text{star}} = 257.22 \cdot \frac{\text{KN}}{\text{m}} > V_u = 223.372 \cdot \frac{\text{KN}}{\text{m}}$$

$V_{\text{star}} > V_u$  therefore bearing capacity OK for earthquake case.

#### Step 5. Wall sliding (earthquake case)

The sliding analysis is carried out with reference to the model shown in Figure X.2. The passive soil resistance in front of the toe of the wall was neglected because of possible desiccation and disturbance. The adhesion underneath the footing is assumed to be the full undrained shear strength of the soil (e.g. concrete poured in contact with rough ground surface).

$$H_{\text{star}} := B_{\text{eff}} \cdot c_a \quad \text{Ultimate sliding resistance}$$

$$H_{\text{star}} = 78.405 \cdot \frac{\text{KN}}{\text{m}} > H_u = 72.547 \cdot \frac{\text{KN}}{\text{m}}$$

**Other issues**

The external stability (global stability) of the wall may need to be checked in certain cases, for example where the ground in front of the retaining wall is sloping away. Also where there is weak ground below or in front of the toe of the wall.

### Drained Bearing Capacity Shallow Footing - Vesic

$$\begin{aligned}
 B &:= B_{\text{eff}} & L &:= 30\text{-m} & D &:= 0.2\text{-m} & & \text{Footing dimensions (effective)} \\
 \beta &:= 0\text{-deg} & & & & & & \text{Ground slope in front of footing} \\
 c_u &:= 0 \cdot \frac{\text{KN}}{\text{m}^2} & & & & & & \text{Soil cohesion} \\
 c_a &:= 1\text{-c} & & & & & & \text{Adhesion (underside of footing)} \\
 q &:= \gamma \cdot D & & & & & & \text{Surcharge}
 \end{aligned}$$

$$N_q := e^{\pi \cdot \tan(\phi)} \cdot \left( \tan\left(\frac{\phi}{2} + \frac{\pi}{4}\right) \right)^2 \quad N_c := (N_q - 1) \cdot \cot(\phi) \quad N_\gamma := 2 \cdot (N_q + 1) \cdot \tan(\phi)$$

$$N_q = 18.401 \quad N_c = 30.14 \quad N_\gamma = 22.402$$

Shape factors

$$\lambda_{cs} := 1 + \frac{B \cdot N_q}{L \cdot N_c} \quad \lambda_{\gamma s} := 1 - 0.4 \cdot \frac{B}{L} \quad \lambda_{qs} := 1 + \frac{B \cdot \tan(\phi)}{L}$$

$$\lambda_{cs} = 1.043 \quad \lambda_{\gamma s} = 0.972 \quad \lambda_{qs} = 1.041$$

Depth factors ( $D < B$ )

$$\lambda_{qd} := 1 + 2 \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 \cdot \frac{D}{B} \quad \lambda_{cd} := \lambda_{qd} - \frac{1 - \lambda_{qd}}{N_c \cdot \tan(\phi)}$$

$$\lambda_{cd} = 1.029 \quad \lambda_{qd} = 1.027 \quad \lambda_{\gamma d} := 1$$

Load inclination factors (loading parallel to B)

$$n_B := \frac{2 + \frac{B}{L}}{1 + \frac{B}{L}} \quad n_B = 1.934$$

$$\lambda_{qi} := \left( 1 - \frac{H_u \cdot L}{V_u \cdot L + L \cdot B \cdot c_a \cdot \cot(\phi)} \right)^{n_B} \quad \lambda_{qi} = 0.659$$

$$\lambda_{\gamma i} := \left( 1 - \frac{H_u \cdot L}{V_u \cdot L + L \cdot B \cdot c_a \cdot \cot(\phi)} \right)^{n_B + 1} \quad \lambda_{\gamma i} = 0.531$$

$$\lambda_{ci} := \lambda_{qi} - \frac{1 - \lambda_{qi}}{N_c \cdot \tan(\phi)} \quad \lambda_{ci} = 0.639$$

Ground inclination factors (see diagram)

$$\lambda_{qg} := (1 - \tan(\beta))^2 \quad \lambda_{cg} := \lambda_{qg} - \frac{1 - \lambda_{qg}}{N_c \cdot \tan(\phi)} \quad \lambda_{\gamma g} := \lambda_{qg}$$

$$\lambda_{qg} = 1 \quad \lambda_{cg} = 1 \quad \lambda_{\gamma g} = 1$$

Base tilt factors (see diagram)

$$\lambda_{qt} := (1 - \eta \cdot \tan(\phi))^2 \quad \lambda_{ct} := \lambda_{qt} - \frac{1 - \lambda_{qt}}{N_c \cdot \tan(\phi)} \quad \lambda_{\gamma t} := \lambda_{qt}$$

$$\lambda_{qt} = 0.737 \quad \lambda_{ct} = 0.722 \quad \lambda_{\gamma t} = 0.737$$

Ultimate bearing pressure

$$q_u := c \cdot \lambda_{cs} \cdot \lambda_{cd} \cdot \lambda_{ci} \cdot \lambda_{cg} \cdot \lambda_{ct} \cdot N_c + q \cdot \lambda_{qs} \cdot \lambda_{qd} \cdot \lambda_{qi} \cdot \lambda_{qg} \cdot \lambda_{qt} \cdot N_q + \frac{1}{2} \cdot \gamma \cdot B \cdot \lambda_{\gamma s} \cdot \lambda_{\gamma d} \cdot \lambda_{\gamma i} \cdot \lambda_{\gamma g} \cdot \lambda_{\gamma t} \cdot N_\gamma$$

$$q_u = 197.333 \cdot \frac{\text{KN}}{\text{m}^2} \quad \text{Ultimate bearing pressure}$$

$$V_{\text{star}} := q_u \cdot B_{\text{eff}} \cdot \Phi_{bc} \quad V_{\text{star}} = 209.774 \cdot \frac{\text{KN}}{\text{m}} \quad V_u = 183.882 \cdot \frac{\text{KN}}{\text{m}}$$

### Undrained Bearing Capacity Shallow Footing - Vesic

$$S_u := 50 \cdot \frac{\text{KN}}{\text{m}^2} \quad c_u := S_u \quad \text{Cohesion = undrained shear strength}$$

$$c_a := 1.0 \cdot S_u \quad \text{Adhesion (underside of footing)}$$

$$B := B_{\text{eff}} \quad L := 30\text{-m} \quad \text{Dimensions of footing}$$

$$D := 0.2\text{-m} \quad \text{Depth of footing}$$

$$q := \gamma \cdot D \quad \text{Surcharge}$$

$$\beta := 0\text{-deg} \quad \text{Ground slope in front of footing}$$

$$N_c := 5.14 \quad N_q := 1 \quad N_\gamma := 0 \quad \text{Bearing capacity factors}$$

Shape factors

$$\lambda_{cs} := 1 + \frac{B \cdot N_q}{L \cdot N_c} \quad \lambda_{\gamma s} := 1 - 0.4 \cdot \frac{B}{L} \quad \lambda_{qs} := 1 + \frac{B \cdot \tan(\phi)}{L} \quad \lambda_{cs} = 1.0 \cdot \lambda_{\gamma s} = 0.979 \quad \lambda_{qs} = 1.03$$

Depth factors

$$\lambda_{qd} := 1 + 2 \cdot \tan(\phi) \cdot (1 - \sin(\phi)) \cdot \frac{D}{B} \quad \lambda_{cd} := 1 + 0.4 \cdot \frac{D}{B} \quad \lambda_{\gamma d} := 1 \quad \lambda_{qd} = 1.037 \quad \lambda_{cd} = 1.051$$

Load inclination factors (loading parallel to B)

$$n := \frac{2 + \frac{B}{L}}{1 + \frac{B}{L}} \quad \lambda_{ci} := 1 - \frac{n \cdot H_u \cdot L}{c_a \cdot N_c \cdot B \cdot L} \quad \lambda_{ci} = 0.649 \quad \lambda_{\gamma i} := 1 \quad \lambda_{qi} := 1$$

Ground inclination factors

$$\lambda_{cg} := 1 - \frac{2 \cdot \beta}{\pi + 2} \quad \lambda_{qg} := (1 - \tan(\beta))^2 \quad \lambda_{\gamma g} := \lambda_{qg} \quad \lambda_{qg} = 1 \quad \lambda_{cg} = 1$$

$$N_{\gamma\beta} := -2 \cdot \sin(\beta) \quad N_\gamma = 0$$

Base tilt factors

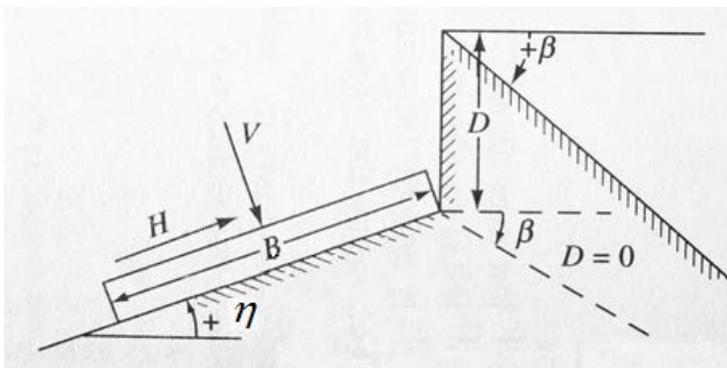
$$\lambda_{ct} := 1 - \frac{2 \cdot \eta}{\pi + 2} \quad \lambda_{ct} = 0.905 \quad \lambda_{\gamma t} := 1 \quad \lambda_{qt} := 1$$

Ultimate bearing pressure

$$q_u := c \cdot \lambda_{cs} \cdot \lambda_{cd} \cdot \lambda_{ci} \cdot \lambda_{cg} \cdot \lambda_{ct} \cdot N_c + q \cdot \lambda_{qs} \cdot \lambda_{qd} \cdot \lambda_{qi} \cdot \lambda_{qg} \cdot \lambda_{qt} \cdot N_q + \frac{1}{2} \cdot \gamma \cdot B \cdot \lambda_{\gamma s} \cdot \lambda_{\gamma d} \cdot \lambda_{\gamma i} \cdot \lambda_{\gamma g} \cdot \lambda_{\gamma t} \cdot N_\gamma$$

$$q_u = 164.032 \cdot \frac{\text{KN}}{\text{m}^2} \quad \text{Ultimate bearing pressure}$$

$$V_{\text{star}} := q_u \cdot B_{\text{eff}} \quad V_{\text{star}} = 257.22 \cdot \frac{\text{KN}}{\text{m}} > V_u = 223.372 \cdot \frac{\text{KN}}{\text{m}}$$



[Source: Bowles 1997]

## References

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